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ESTIMATING THE TOP TAIL OF THE FAMILY WEALTH DISTRIBUTION IN CANADA



OFFICE OF THE PARLIAMENTARY BUDGET OFFICER
BUREAU DU DIRECTEUR PARLEMENTAIRE DU BUDGET

The Parliamentary Budget Officer (PBO) supports Parliament by providing economic and financial analysis for the purposes of raising the quality of parliamentary debate and promoting greater budget transparency and accountability.

PBO has developed a modelling approach to estimate the top tail of the family wealth distribution in Canada. The modelling approach produces a new micro database of high-net-worth families to undertake analytical and costing work. This report describes the approach to constructing the database and showcases its analytical capabilities.

PBO wishes to acknowledge Professor Jim Davies, who provided valuable technical clarifications related to estimating the top tail of the family wealth distribution, and officials from Statistics Canada's Survey of Financial Security (SFS) Team, who answered many questions related to the SFS.

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Executive Summary

The Parliamentary Budget Officer (PBO) has developed a modelling approach to estimate the top tail of the family wealth distribution in Canada. Its main purpose is to address underreported and missing data of high-net-worth families in the Survey of Financial Security Public Use Microdata File (SFS PUMF). Drawing on the National Balance Sheet Accounts (NBSA), the modelling recalibrates the SFS PUMF to add a synthetic dataset of families with wealth over \$3 million.

This modelling work produced a new analytical resource, the *High-net-worth Family Database* (HFD). HFD enables PBO to produce cost estimates and analysis of measures affecting Canadian families with wealth in the millions and billions of dollars.

Using HFD, PBO finds that Canada's wealthiest families have significantly more wealth than recorded in the SFS PUMF. HFD increases the wealth share of the top 1 per cent of families by 12 percentage points compared with the SFS PUMF (Table ES-1). The discrepancy is likely due to sampling and non-sampling errors, especially higher survey non-response among high-net-worth families, in the SFS.

Table ES-1 Family wealth distribution, SFS PUMF and HFD, by selected quantiles, Canada, 2016

Family wealth quantile	SFS PUMF	HFD
	Share of total wealth (per cent)	Share of total wealth (per cent)
Top 0.01%	0.4	5.6
Top 0.1%	3.1	12.1
Top 0.5%	9.2	20.5
Top 1%	13.7	25.6
Top 5%	33.0	43.4
Top 10%	47.6	56.4
Top 20%	67.2	73.5
Middle 40%	30.5	25.3
Bottom 40%	2.3	1.2

Sources: PBO calculations of the SFS PUMF; PBO High-net-worth Family Database

This report describes the modelling approach used to produce the synthetic dataset of high-net-worth families, to incorporate it into the SFS PUMF, and to align aggregate values in the combined dataset with those in the NBSA. It will serve as a reference for future PBO work on the topic as it arises.

HFD was constructed using publicly-available data. Additional documentation is available upon request.

1. Introduction

During the 2019 federal election, the Parliamentary Budget Office (PBO) estimated the financial cost of electoral proposals of political parties upon request.¹ One such request was made to estimate the fiscal revenues of an annual tax on the net wealth of high-net-worth families above \$20 million.²

PBO faced a key barrier to meet the request: The lack of a publicly available micro database that reliably assesses high-net-worth families in Canada. For example, Statistics Canada's principal family wealth microdata product, the Survey of Financial Security Public Use Microdata File (SFS PUMF), reports families with wealth up to only \$27 million. By contrast, the lowest entry on Canadian Business magazine's list of the 100 "Richest People" had a wealth of \$875 million.

To address the data gap, PBO developed a modelling approach to reliably estimate the top tail of the family wealth distribution in Canada. This approach consisted of adapting a straight-forward Pareto interpolation technique in Bach et al. (2014) and Saez and Zucman (2019). The technique creates a synthetic dataset bridging wealth microdata from the SFS PUMF and the Canadian Business (CB) magazine's Richest People List. This synthetic dataset enabled PBO to fulfil the electoral costing request with a two-page [cost estimate](#), published in September 2019.

Since the federal election, PBO decided to build on that work and develop a functional analytical tool of high-net-worth families. To do so, the modelling approach used in the election underwent several refinements. The most significant of these was applying a modified ordinary least squares (OLS) regression and iterative calibration procedure developed in Vermeulen (2016) and (2018). The refined approach aligns the aggregate asset, liabilities, and net worth values in the re-estimated family wealth distribution with those in the National Balance Sheet Accounts (NBSA). As a result of these refinements, what was reported in PBO's electoral proposal cost estimate is not directly comparable with the results in this report.

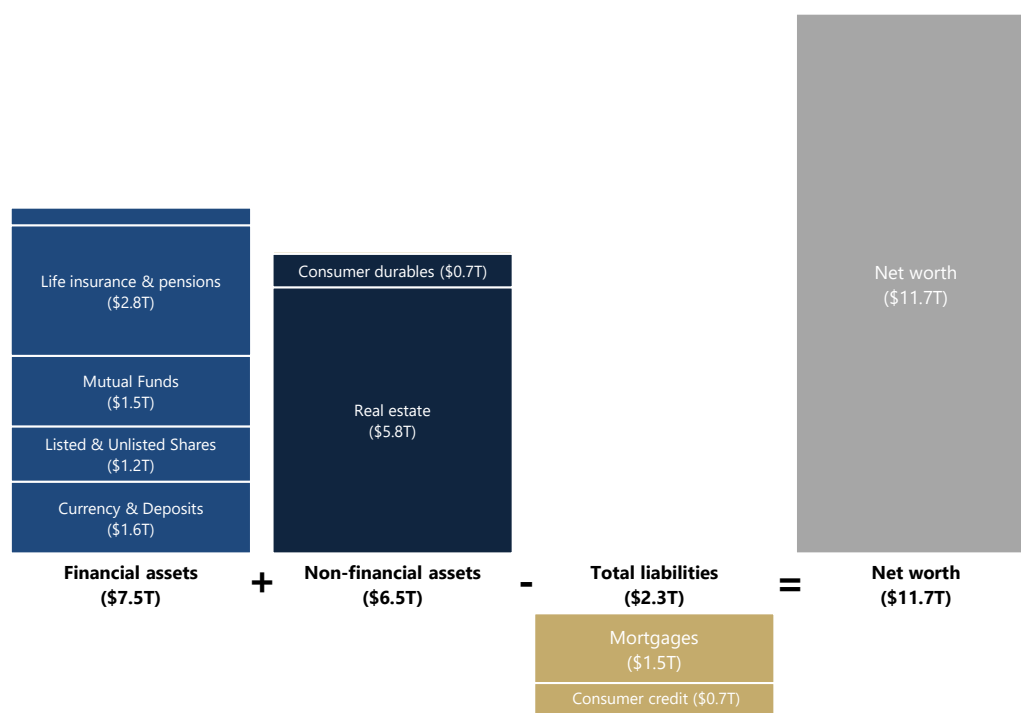
The ultimate product from this modelling work is the *High-net-worth Family Database* (HFD). HFD was constructed using publicly available data from year-end 2016, the most recent date all sources reported data. It will be used to undertake analytical and costing work on high-net-worth families as it arises.

To showcase the kind of analytical work that is feasible using HFD, summary statistics from the database are presented in Section 4 and Appendix B of the report. These results are for illustrative purposes and may differ from analysis of a specific measure using HFD.

2. Measuring family wealth in Canada

For the purposes of this report, PBO measured family wealth in terms of marketable net worth: the amount of money left to a family if it liquidates all its financial and non-financial assets and paid off all its liabilities.^{3,4} Canadian families collectively hold significant wealth. According to Statistics Canada's National Balance Sheet Accounts (NBSA), which record the stock of assets, liabilities and net worth for each institutional sector, at the end of 2019 Canada's household sector held \$11.7 trillion in total net worth. That figure is approximately five times larger than Canada's annual GDP.⁵ Real estate (\$5.8 trillion) and mortgages on that real estate (\$1.5 trillion) are the single largest asset and liabilities categories, respectively (Figure 2-1).

Figure 2-1 Household assets, liabilities and net worth, Canada, 2019 Q4



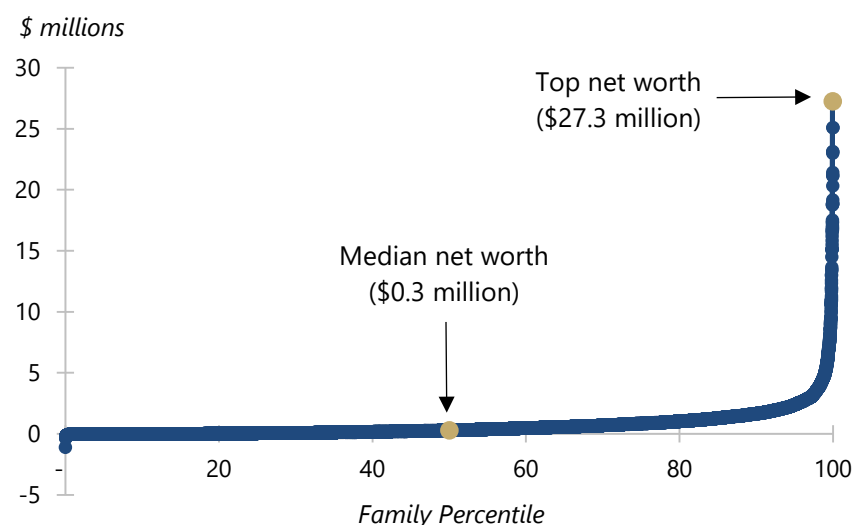
Source: PBO calculations of Statistics Canada Table 36-10-0580-01 (National Balance Sheet Accounts for the household sector, 2019 Q4)

The distribution of wealth among households is heavily skewed toward the wealthiest families.⁶ In Canada, a small proportion of families at the top of the distribution possess net worth that is orders of magnitude higher than the country's median net worth (Figure 2-2). The high concentration of wealth among a small number of families makes it difficult to reliably measure wealth at the very top of the distribution. This difficulty is evident in

the Survey of Financial Security Public Use Microdata File (SFS PUMF), Statistics Canada's national survey to measure Canadians' net worth. The wealthiest family observed in the 2016 SFS PUMF had a net worth of only \$27 million;⁷ the survey did not report any wealthier families, for several potential reasons (Box 2-1).

Figure 2-2

Distribution of family net worth, Survey of Financial Security Public Use Microdata File, 2016



Source: PBO calculations using the 2016 SFS PUMF

There are at least four general approaches that can be taken to improve estimates of the top tail of the family wealth distribution. The first involves compiling dossiers on each high-net-worth family, much like the Forbes World's Billionaires list. The second uses individual income tax returns to capitalize the incomes reported by taxpayers. The third uses estate tax records to back out the wealth recorded by the deceased and makes certain assumptions about how the recorded wealth of the deceased relates to the actual wealth of the living. The fourth consists of adjusting the family wealth distribution in national surveys like the SFS PUMF using data from other sources. This last approach is PBO's preferred approach and is further developed in the next section.

Box 2-1 Limitations of national wealth surveys in measuring high-net-worth families

There are several plausible reasons national wealth surveys, like Canada's SFS, are limited in measuring and analyzing high-net-worth families.

Surveys may be subject to **sampling errors** if the surveyed sample is not representative of the population, including at the top of the family wealth distribution.

Response errors, where families inaccurately report, willingly or not, the value of their assets and liabilities, may bias estimates for high-net-worth families.

Certain large asset and liabilities values in the SFS PUMF are also subject to **top-coding**, where they are replaced with a maximum value. While this procedure ensures the confidentiality of released data, it also reduces top wealth shares (see Appendix A.3).

The most impactful limitation may be **differential unit non-response**, the tendency of high-net-worth families to be less likely to participate in surveys. If high-net-worth families are undersampled and the survey weights of those that are sampled are not adequately scaled upwards, top wealth shares will be underestimated.

While Statistics Canada reports the overall response rate (70.3 per cent for the 2016 SFS), little is publicly-known about the incidence of differential unit non-response in the SFS. There is evidence from the U.S. of a positive correlation between wealth and the rate of unit non-response in its main wealth survey, the Survey of Consumer Finances (Kennickell & Woodburn, 1997)

Statistics Canada attempts to address differential unit non-response among high-net-worth families by oversampling geographic areas known to have higher income and believed to have higher wealth (Statistics Canada, 2018a). However, similar approaches to oversample high-net-worth families using geographic or income-stratified geographic information in several European countries have been shown to be of limited effectiveness in accurately measuring the wealth of high-net-worth families (Vermeulen, 2018).

3. Database construction

PBO's *High-net-worth Family Database* (HFD) was constructed using data from three sources:

1. **The Survey of Financial Security Public Use Microdata File.**⁸ The SFS PUMF is Canada's national net worth survey. Statistics Canada surveys a representative sample of over 12,000 resident economic families on their major financial and non-financial assets and debts.⁹ HFD uses the most recently-published iteration of the SFS PUMF, from 2016.
2. **The National Balance Sheet Accounts.** The NBSA aggregate the individual balance sheets of households across the economy and reports their aggregate financial assets, non-financial assets, liabilities, and ultimately net worth.¹⁰ HFD uses NBSA data from 2016 Q4, the date that aligns most closely with the vintages of the SFS PUMF and CB's Richest People List used in the database.¹¹
3. **Canadian Business magazine's Richest People List.** CB conducts journalistic and market research to compile a list of the 100 wealthiest Canadian citizens.¹² HFD uses CB's 2017 Richest People List, which was published in December 2016 and corresponds most closely with the 2016 SFS PUMF.

PBO followed Vermeulen's (2016) elegant approach to address missing and underreported data of high-net-worth families in the SFS PUMF and build HFD. First, the aggregate values of financial assets, non-financial assets, and total liabilities in the SFS PUMF were adjusted to align with the corresponding totals by category in the NBSA. Second, data from CB's Richest People List were added to the SFS PUMF. Third, the resulting joint dataset was used to run a modified OLS regression that would determine the shape of the wealth distribution for the missing and underreporting families and bridge the top of the SFS PUMF and the bottom of the CB Richest People List. Fourth, the results from the modified OLS regression were applied to create a new synthetic dataset of high-net-worth families. Fifth, the synthetic dataset was merged with the joint dataset from the second step. The addition of the synthetic dataset generally creates more assets and liabilities than there are in the NBSA, which leads to sixth step: to reduce (or increase) each of the financial assets, non-financial assets, and total liabilities in the SFS PUMF by an adjustment factor and returning to the second step to repeat the procedure iteratively until the value of the financial assets, non-financial assets, and total liabilities in the final, integrated dataset (combining NBSA-adjusted SFS PUMF, the synthetic dataset, and CB's Richest People List) are equal to those in the NBSA.

The modelling approach used to construct HFD is described in greater detail in Appendix A.

4. Database capabilities

PBO generated summary statistics of HFD to showcase its analytical capabilities.

Using HFD, PBO finds that Canada's wealthiest families have significantly more wealth than recorded in the SFS PUMF. The wealth share of the top 1 per cent of families increases by 12 percentage points in HFD compared with the SFS PUMF (Table 4-1).

Table 4-1 Family wealth distribution, SFS PUMF and HFD, by selected quantiles, Canada, 2016

Family wealth quantile	SFS PUMF	HFD
	Share of total wealth	Share of total wealth
	(per cent)	(per cent)
Top 0.01%	0.4	5.6
Top 0.1%	3.1	12.1
Top 0.5%	9.2	20.5
Top 1%	13.7	25.6
Top 5%	33.0	43.4
Top 10%	47.6	56.4
Top 20%	67.2	73.5
Middle 40%	30.5	25.3
Bottom 40%	2.3	1.2

Sources: PBO calculations of the SFS PUMF; PBO High-net-worth Family Database

Appendix B presents additional summary statistics for year-end 2016, HFD's 'base' period when each of its sources most recently reported data.

Analyzing high-net-worth families in subsequent periods requires making certain assumptions about the evolution of families and their wealth since the end of 2016. To illustrate the kinds of assumptions required to bring HFD forward, PBO also generated summary statistics on high-net-worth families for year-end 2019. PBO assumed that, since 2016:

- The composition of families (number of people, age, etc.) has remained constant across the wealth distribution;¹³

- The number of families has grown at the same rate as the number of individuals, and this growth has been uniform across the wealth distribution;¹⁴
- Aggregate financial assets, non-financial assets, and total liabilities have grown at the same rate as indicated in the NBSA, and this growth has been proportional across the family wealth distribution.

Following these assumptions, PBO applied two adjustments to HFD. First, the weight of each observation was increased by growth rate of Canada's population between 2016 Q4 and 2019 Q4. Second, the financial assets, non-financial assets, and total liabilities of each observation was increased proportionally, until their aggregate totals matched those in the NBSA in 2019 Q4.

The resulting summary statistics are presented in Tables 4-2 and 4-3. Both tables highlight the strong concentration of wealth among Canada's high-net-worth families.

Table 4-2 Family wealth distribution, by selected quantiles, Canada, 2019

Family wealth quantile	Wealth threshold	Number of families	Total wealth	Share of total wealth
	(\$ millions)	(thousands)	(\$ billions)	(per cent)
Top 0.01%	143.1	1.6	654	5.6
Top 0.1%	29.3	16.0	1,427	12.2
Top 0.5%	9.7	79.7	2,410	20.6
Top 1%	6.1	159.3	3,010	25.7
Top 5%	2.3	796.7	5,107	43.7
Top 10%	1.6	1,593.5	6,629	56.7
Top 20%	1.0	3,186.9	8,633	73.8
Middle 40%	0.1-1.0	6,373.8	2,932	25.1
Bottom 40%	under 0.1	6,373.8	132	1.1

Source: PBO High-net-worth Family Database; PBO calculations based on Statistics Canada's Quarterly Demographic Estimates and the NBSA

Table 4-3 Wealth distribution, by selected wealth thresholds, Canada, 2019

Family wealth threshold	Number of families	Total wealth	Share of total wealth
Families with wealth above:	(thousands)	(\$ billions)	(per cent)
\$1 billion	0.1	221	1.9
\$500 million	0.2	333	2.8
\$250 million	0.7	488	4.2
\$100 million	2.7	785	6.7
\$50 million	7.2	1,097	9.4
\$25 million	19.4	1,525	13.0
\$10 million	76.3	2,377	20.3
\$5 million	206.6	3,271	28.0
\$2.5 million	699.1	4,871	41.6
\$1 million	3,123.7	8,570	73.3

Source: PBO High-net-worth Family Database; PBO calculations based on Statistics Canada's Quarterly Demographic Estimates and the NBSA

Appendix A: Modelling approach and assumptions

A.1 Initial data alignment

PBO performed an initial adjustment to the SFS PUMF microdata so that the aggregate values of assets, liabilities, and net worth aligned with the corresponding totals by category for the household sector in the NBSA.

While the SFS PUMF and the NBSA both estimate household net worth, there are procedural and conceptual distinctions between the two sources that lead to slightly different estimates.¹⁵ Most obviously, the SFS PUMF is derived from a survey with confidence intervals on its estimates; the NBSA measure stocks and flows in capital and financial accounts – but because certain household categories are calculated as residuals from other sectors, the NBSA have a margin of error of their own. The SFS does not sample the territories and certain population groups representing two per cent of the population. Certain assets and liabilities are also measured differently. For example, the NBSA do not record the value of collectibles such as art work; the two sources measure credit card debt differently, the main reason Statistics Canada (2019a) identifies for under-coverage of total liabilities in the SFS PUMF (Table A1-1).

Table A1-1 Concordance between the SFS PUMF and the NBSA Household Sector, 2016

	SFS PUMF (\$ billions)	NBSA (\$ billions)	Coverage (SFS/NBSA)
Financial assets	5,845	6,468	0.904
Non-financial assets	6,193	5,934	1.043
Total liabilities	1,751	2,062	0.850
Net worth	10,287	10,339	0.995

Sources: PBO calculations of the 2016 SFS and Statistics Canada Table 36-10-0580-01

Notes: NBSA totals reflect results for 2016 Q4.

Business equity was counted as a financial asset.

Totals may not add due to rounding.

Nevertheless, there are several reasons for which it is desirable to bring the SFS PUMF into alignment with the NBSA. Alignment can compensate for underreporting in national wealth surveys (Vermeulen, 2016). Unlike the SFS and its predecessor, the Survey of Consumer Finances (SCF), the NBSA have

been estimated on a consistent basis over time (Davies and Di Matteo, 2020). This consistency allows for better comparison of the family wealth distribution estimates going back in time. The NBSA are also estimated and released more frequently (quarterly) than the SFS (triennially). The higher frequency provides opportunities to update estimates in non-survey years of the SFS. Finally, alignment with the household sector of the NBSA permits analyses of the overall position of households relative to other economic sectors included in the NBSA (Statistics Canada, 2019a). For some of these same reasons, Statistics Canada also performs alignment between the SFS and the NBSA in its Distributions of Household Economic Accounts (DHEA) dataset.

To bring the SFS PUMF into alignment with the NBSA, PBO first classified each asset and debt variable from the SFS PUMF into three large categories: financial assets;¹⁶ non-financial assets; and total liabilities.¹⁷ For each category, PBO calculated an adjustment factor as the inverse of the “coverage” calculation in Table A1-1. We increased (decreased) the financial assets, non-financial assets, and total liabilities values for each family in the SFS PUMF by the relevant adjustment factor. Since each family has a unique portfolio of assets and liabilities, their net worth varies differently with this adjustment procedure.¹⁸

A.2 Rich list data incorporation

The next procedure consisted of adding wealth data from a rich list to the NBSA-adjusted SFS PUMF.

The motivation to augment the SFS PUMF with rich list data is to improve the accuracy of the subsequent regression analysis that estimates Pareto parameters used in the imputation of the missing and underreported high-net-worth families.¹⁹ Vermeulen (2018) demonstrates that the addition of even a small number of entries from a rich list significantly improves the accuracy of interpolated top tail estimates, enough that there is almost no estimation bias.²⁰

In Canada there are two prominent, publicly available rich lists: the Forbes list of the world’s billionaires, which includes Canadian entries; and Canadian Business (CB) magazine’s Richest People List. PBO elected the latter for HFD, following Davies and Di Matteo (2020). They note that CB’s list contains billionaires missing in the Forbes list and includes entries below Forbes’ US\$1 billion cut-off.²¹

Before they could be added to the NBSA-adjusted SFS PUMF, data from CB’s Richest People List required adjustment.²²

Unlike the SFS PUMF, CB includes non-resident Canadians in its accounting of the 100 Richest Canadians. As a result, PBO dropped non-resident Canadians from the CB dataset, similar to MacDonald (2018).

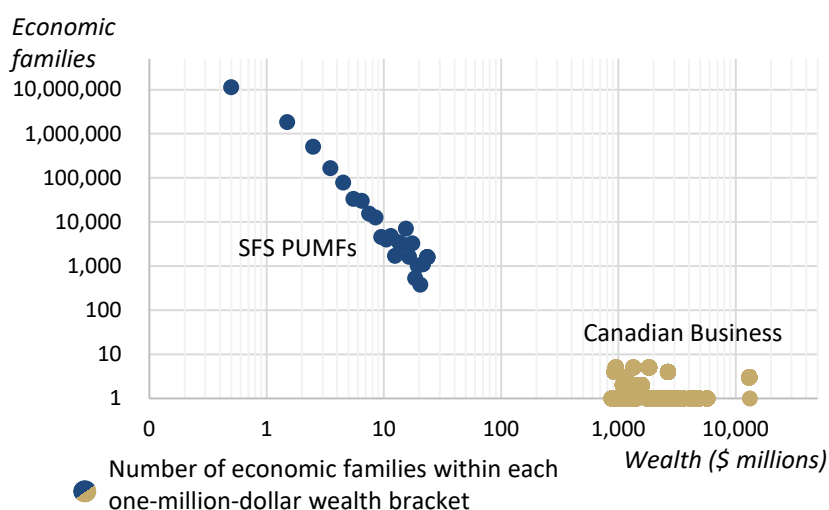
In addition, several CB entries refer to extended families comprising multiple family units. These include entries entitled “family”, “brothers”, and “estate”, or that otherwise listed multiple people who were not married. By contrast, the SFS PUMF family unit consists of economic families and persons not in an economic family (unattached individuals).

PBO developed an approach to split extended families in the CB into constituent economic families. We used public sources to identify the generation(s) controlling the family wealth. Each sibling (and cousin, if applicable) within the controlling generation(s), as well as their living parent(s) (and uncles and aunts, if applicable), was treated as a unique economic family. We assumed that the extended family’s reported wealth resides exclusively and entirely within the identified constituent economic families. We also assumed that the extended family’s wealth is divided evenly among its constituent economic families. Finally, we dropped all split entries that fell below the lowest entry (\$875 million) on the CB list. This final procedure ensured that the top of the wealth tail, above \$875 million, comprised of a complete population of families above that level to avoid bias in the subsequent regression analysis to estimate Pareto parameters.

Following this splitting procedure, the cleaned CB dataset included 80 resident economic families. Each held a wealth of at least \$875 million, and collectively they held \$197 billion in wealth.

PBO added the cleaned CB data to the NBSA-adjusted SFS PUMF, creating a *joint dataset* (see Figure A2-1). Each CB observation was assigned a weight of 1, reflecting that each observation represents a one family unit.

Figure A2-1 Family wealth distribution in the joint dataset,²³ 2016



Sources: PBO calculations of the 2016 SFS PUMF and Canadian Business' Richest People List, 2017

A.3 Pareto interpolation

PBO used the *joint dataset* to impute the missing and underreported high-net-worth families. To do so, PBO referred to the modified OLS regression approach for complex survey designs in Vermeulen (2018). The resulting estimated Pareto parameters were then used to interpolate the missing and underreported high-net-worth families.

A key assumption for this imputation procedure is that the top of the family wealth tail exhibits a Pareto distribution. This assumption has been widely applied in the literature on wealth distributions, including in Canada. Davies and Shorrocks (1999) characterize the notion that the top wealth tail follows a Pareto distribution as an “enduring feature” of the wealth distribution. Brzozowski et al. (2010) assume that the top decile of the SFS is Pareto-distributed in their comparison of different statistical methods to impute top-coded observations into the SFS PUMF. Ogwang (2011) finds that CB's Richest People List from 1999 to 2008 displays Pareto power law²⁴ behaviour using modified OLS and MLE estimation methods. Davies and Di Matteo (2020) assume that the top wealth tail follows a Pareto distribution in their analysis of the evolution of top family wealth shares in Canada between 1892 and 2016.

Vermeulen (2018) notes another key assumption: that the national wealth survey and rich list datasets “are consistent with the same Pareto

distribution". PBO makes that assumption, but it's a cautious one for two reasons. The first is due to the reliability and substance of documentation available on CB's methodology. The most recent CB methodology that PBO could locate dates from 2006.²⁵ The methodology provides useful information about CB's approach, at least for that year. The methodology indicates that at least certain debts (privately-owned companies, real estate) are ascertained or estimated, and deducted from total assets. However, the methodology also states that "intentionally [...] conservative estimates" are used to value private investments and that "it's safe to assume the Rich 100 are worth more than the stated amount" (Canadian Business, 2006). Davies and Di Matteo (2020) note that the problems of rich list data compilation are reduced by the scrutiny the lists attract and the refinements the lists undergo as they are repeated annually (CB has been compiling a rich list since 1999). While it's reasonable to assume that CB approximates the wealth of the highest-net-worth Canadians, it's unclear what, if any, bias there may be in its dataset.

The second note of caution in assuming the *joint dataset* lies on the same Pareto distribution is due to top-coding in the SFS PUMF. The SFS PUMF is top-coded such that a certain number of the largest values on some of the assets and debts are replaced with a maximum value to ensure the confidentiality of each observation disclosed in public use files. However, it also reduces the wealth of the top families in the SFS PUMF relative to SFS data available at a Research Data Centre (RDC), which is not top-coded.²⁶ Brzozowski et al. (2010) reported that the wealth share of the top 1 percent of families was approximately 1.5 percentage points lower in the 1999 SFS PUMF (13.2 percent) than in the 1999 SFS RDC data (15.7 percent). The degree of top-coding in the 2016 SFS PUMF is not reported publicly, and PBO has not analyzed the extent of top-coding or its potential bias on the Pareto estimates. In theory, this potential bias is reduced by estimating the Pareto parameters over a sufficiently large segment of the top tail of the *joint dataset*; a larger segment should include, proportionally, fewer top-coded families, reducing the potential bias those families could introduce.

To apply Vermeulen's (2018) regression approach, PBO first isolated a subset of observations i in the *joint dataset* with wealth $w_i \geq w_{min}$ over which the regression would be run. The choice of an appropriate or even a best-fit w_{min} is unclear and often determined case by case.²⁷ In Vermeulen's (2018) re-estimation of top wealth shares in 10 European countries and the U.S., the choice of w_{min} depends, in part, on the method used in each country's national wealth survey to oversample high-net-worth families, who are less likely to respond to such surveys. Countries that oversample using individual information, such as income tax information (the U.S.) or taxable wealth information (Spain, France), were each tested with w_{min} ranging from €500,000 to €10 million. By contrast, countries that oversample using income-stratified geographic information (Germany, Belgium), geographic information only (Austria, Portugal), or no oversampling at all (Italy,

Netherlands) were each tested with w_{min} ranging from €500,000 to only €2 million. In those countries, there were too few observations above thresholds higher than €2 million to accurately estimate Pareto parameters.

Canada's SFS does not appear to use individual information to oversample high-net-worth families. The survey stratifies each province into rural and urban areas. In rural areas, the SFS uses geographic information from the Labour Force Survey area frame to select a multi-stage sample. In urban areas, the SFS uses information from the Socioeconomic indicators File (SEF) T1 Family File (T1FF), such as age and income, to stratify the Address Register into groups of dwellings predicted to have similar wealth (Statistics Canada, 2018b). The urban stratum for the highest wealth represents the top 5 percent of each province.²⁸

PBO thus narrowed the range of appropriate w_{min} in the Canadian context to between \$750,000 and \$3 million, an approximate conversion of the euro values of w_{min} used in Vermeulen (2018) for countries that also use geographic and income-stratified geographic information to oversample high-net-worth families. Vermeulen (2018) and Chakraborty et al. (2019) highlight a trade-off when selecting a specific w_{min} : A lower threshold will increase the sample size for the regression leading to a more reliable Pareto estimation, but at the risk of potentially including observations that do not follow Pareto tail behaviour.

Ultimately, PBO chose the upper-bound of the range of appropriate w_{min} : \$3 million. Compared with national wealth surveys in European countries that oversampled using geographic or income-stratified geographic information, Canada's SFS PUMF has comparatively many more observations at the €2 million / \$3 million threshold.²⁹ The choice of a higher w_{min} permits more observations from the NBSA-adjusted SFS PUMF to be retained post-interpolation while maintaining a robust sample size to undertake the regression estimate of Pareto parameters.

Having chosen a w_{min} , PBO ranked all observations n with wealth $\{w_i, i = 1, \dots, n \mid w_i \geq w_{min}\}$ from the *joint dataset* in descending order of their wealth. Each observation with wealth w_i and weight N_i was defined in terms of \bar{N} , the average weight of all observations with wealth equal or greater than w_{min} , and \bar{N}_{fi} , the average weight of the wealthiest i observations ($\bar{N}_{fi} = \frac{\sum_{j=1}^i N_j}{i}$).

Vermeulen (2018) proposes one final specification to the regression. Gabaix and Ibragimov (2011) found that log-rank-log-size OLS regressions were systematically, strongly biased in small samples. Vermeulen (2018) therefore reduces the rank of each observation in the regression by $\frac{1}{2}$. The modification reduces the bias to a leading order.

The resulting modified OLS regression is described by:

$$\ln((i - 0.5) \frac{\bar{N}_{f_i}}{\bar{N}}) = \ln(n) + \alpha(\ln(w_{min}) - \ln(w_i))$$

The estimated α coefficient from the above regression is the Pareto parameter, which determines the shape of re-estimated top tail of the family wealth distribution. In general, a higher α results in a fatter top tail and a higher concentration of wealth. The estimated α coefficient was applied to a standard Pareto cumulative distribution function over a given wealth interval $\{[w_k, w_i] \mid w_i \geq w_k \geq w_{min}\}$:

$$f(w_i, w_k) = -w_{min}^\alpha * \left(\left(\frac{1}{w_i^\alpha} \right) - \left(\frac{1}{w_k^\alpha} \right) \right)$$

The above cumulative distribution function $f(w_i, w_k)$ yields estimates between 0 and 1 for the probability that a family in the top tail will have wealth between w_k and w_i . Following Chakraborty et al. (2019), cumulative distribution estimates were converted into the number of synthetic families within the wealth interval $[w_k, w_i]$ by multiplying the distribution function $f(w_i, w_k)$ by $n\bar{N}$, the sum of the weights of all n observations with wealth $\{w_i, i = 1, \dots, n \mid w_i \geq w_{min}\}$. Like Davies and Di Matteo (2020), PBO retained the cleaned CB entries without interpolation. The resulting synthetic dataset consists of families with wealth between w_{min} (\$3 million) and w_{cbm} , the wealth of the lowest entry from the cleaned CB dataset (\$875 million).

After replacing observations from the *joint dataset* with wealth $\{w_i, i = 1, \dots, n \mid w_{min} \leq w_i < w_{cbm}\}$ with the synthetic dataset, PBO created an *integrated dataset*. The *integrated dataset* combines families from the NBSA-adjusted SFS PUMF with wealth under w_{min} ; families from the synthetic dataset with wealth between w_{min} and w_{cbm} ; and families from the cleaned CB with wealth equal or higher than w_{cbm} .

A.4 Iterative calibrations

Substituting high-net-worth families in the NBSA-adjusted SFS PUMF with the interpolated synthetic and cleaned CB datasets creates a problem: Families in the new *integrated dataset* possess more aggregate wealth than the household sector in the NBSA.

PBO followed Vermeulen (2016) to implement an iterative calibration procedure that aligns aggregate asset, liabilities, and net worth values in the *integrated dataset* with those in the NBSA.

The first step requires returning to the NBSA-adjusted SFS PUMF (the product of Appendix A.1). For families with wealth $w_i \geq w_{min}$ in the NBSA-adjusted SFS PUMF, PBO calculated three ratios: aggregate financial assets to aggregate net worth; aggregate non-financial assets to aggregate net worth;

and aggregate total liabilities to aggregate net worth. The ratios were then applied uniformly to synthetic and CB families in the *integrated dataset* to divide their wealth into constituent asset and liabilities values.

In the next step, PBO calculated aggregate values for financial assets, non-financial assets, and total liabilities across the entire *integrated dataset*. The aggregate values in the *integrated dataset* were compared with their corresponding values in the NBSA. To the extent that *integrated dataset* aggregate values were higher (lower) than the NBSA, PBO applied a downward (upward) revision to the adjustment factors applied to the original SFS PUMF data in Appendix A.1. From there, PBO re-estimated the Pareto parameters in Appendix A.3 and repeated this adjustment and re-estimation procedure iteratively until the aggregate values of financial assets, non-financial assets, and total liabilities were aligned to their corresponding values in the NBSA. This procedure typically required several repetitions to produce NBSA-calibrated values for all assets and liabilities. The final, calibrated value of the α parameter, which determines the shape of the family wealth distribution, was 1.45.³⁰

The final adjustment factors applied to SFS PUMF data to bring the *integrated dataset* into alignment with the NBSA are presented in Table A4-1. Altogether, financial and non-financial assets were reduced by 5.8 percent and 13.0 percent, respectively. Total liabilities were adjusted up by 12.8 percent, reflecting the significantly lower reported liabilities in the SFS PUMF compared with the NBSA.

Table A4-1 Adjustment factors applied to the SFS PUMF to align aggregate asset and liabilities values in the integrated dataset with the NBSA

	Initial alignment Appendix A.1	Iterative calibrations Appendix A.4	Overall (A.1*A.4)
Financial assets	1.106	0.852	0.942
Non-financial assets	0.959	0.907	0.870
Total liabilities	1.176	0.959	1.128

Source: PBO calculations

The iterative calibration procedure was repeated until aggregate values for assets and debts were within 0.00001 per cent of the corresponding values in the NBSA.

PBO applied a very small, proportional adjustment to the financial assets, non-financial assets, and total liabilities of all families in the *integrated dataset* to fully align their aggregate values with those in the NBSA.

The resulting dataset is the *High-net-worth Family Database* (HFD).

Appendix B: Summary statistics

Tables B-1 and B-2 present summary statistics from HFD for its base year 2016. The HFD's results are comparable to other precedents in the literature: wealth shares in Table B-1 are comparable to Davies and Di Matteo (2020); the number and wealth of families in Table B-2 are similar to Wealth-X (2017); and the HFD's overall finding of significant upward revisions to top wealth shares relative to national wealth surveys dovetails results in Bach et al. (2015), Vermeulen (2016) and (2018), and Davies and Di Matteo (2020).

For reference in interpreting the summary statistics, the calibrated HFD represents approximately 15,349,000 families that collectively possess \$10.3 trillion in wealth.

Table B-1 Family wealth distribution, by selected quantiles, Canada, 2016

Family wealth quantile	Wealth threshold	Number of families	Total wealth	Share of total wealth
	(\$ millions)	(thousands)	(\$ billions)	(per cent)
Top 0.01%	130.5	1.5	574	5.6
Top 0.1%	26.7	15.3	1,254	12.1
Top 0.5%	8.9	76.7	2,117	20.5
Top 1%	5.5	153.4	2,644	25.6
Top 5%	2.1	767.5	4,488	43.4
Top 10%	1.4	1,534.9	5,829	56.4
Top 20%	0.9	3,069.9	7,599	73.5
Middle 40%	0.1-0.9	6,139.7	2,613	25.3
Bottom 40%	under 0.1	6,139.7	128	1.2

Source: PBO High-net-worth Family Database

Table B-2 Family wealth distribution, by selected wealth thresholds, Canada, 2016

Wealth threshold	Number of families	Total wealth	Share of total wealth
Families with wealth above:	(thousands)	(\$ billions)	(per cent)
\$1 billion	0.1	184	1.8
\$500 million	0.2	277	2.7
\$250 million	0.6	408	3.9
\$100 million	2.2	656	6.3
\$50 million	6.2	925	8.9
\$25 million	16.7	1,287	12.5
\$10 million	63.7	1,994	19.3
\$5 million	173.8	2,751	26.6
\$2.5 million	549.5	3,983	38.5
\$1 million	2,699.0	7,246	70.1

Source: PBO High-net-worth Family Database

Appendix C: Future database development

Future work on HFD will be guided by topics of relevance to parliamentarians, the availability of new data sources, and the evolution of the academic literature on measuring top wealth shares.

PBO wishes to verify whether top-coding in the SFS PUMF introduces bias to the estimation of Pareto parameters. This analysis can be done by constructing HFD using SFS data from a Statistics Canada Research Data Centre (RDC), where observations are not top-coded, and comparing the SFS PUMF and SFS RDC versions of HFD.

Statistics Canada collected data for its next iteration of the SFS between September and December 2019 (Statistics Canada, 2019b). It's unclear when the new public use microdata file will be available. PBO plans to adapt HFD to the most recent publicly available version of the SFS, which will be conducted triennially going forward.

Future database development may also have to contend with the potential loss of an existing data source. The rich list used to construct HFD came from CB's Richest People 2017, which corresponds to data from 2016. While CB published a list for 2018 (corresponding to data from 2017), PBO has not been able to locate a 2019 publication of this list. If CB has discontinued publication of an annual rich list, PBO will consider alternative rich lists, such as the Forbes World's Billionaires List, to update HFD.

Finally, there is the potential for future research to offer opportunities to refine the modelling approach used to construct HFD. Topics of interest include the identification of a best-fit w_{min} , the wealth threshold at which Pareto interpolation begins; a more refined approach to divide the wealth of synthetic high-net-worth families into constituent asset and liabilities categories; the possibility to estimate more granular categories of assets and liabilities of high-net-worth families; and the consideration of incorporating non-marketable forms of wealth in estimates of the top tail of the family wealth distribution. PBO plans to monitor the academic literature for new theories and methodologies that could refine or enhance HFD.

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Notes

¹ Parliamentary Budget Officer (2019b). Over four months leading up to the 2019 federal election, PBO costed over 200 electoral proposal requests from political parties.

² Parliamentary Budget Officer (2019a). More specifically, PBO was requested to estimate the revenues from “introducing an annual net wealth tax on Canadian resident economic families equal to 1% of net wealth above \$20 million” for which “all asset and liabilities will be included in the net wealth tax base, except wealth won in lotteries”.

³ This definition is the same as that in the Survey of Financial Security (Statistics Canada, 2018b) and forms the statistical foundation of PBO’s modelling in this report. For the purposes of this report, the definition applies equally to the terms “net worth” and “wealth”, which are used interchangeably.

⁴ There is an emerging literature on whether to include non-marketable forms wealth in the estimation of household wealth and wealth shares and, if so, how. Weil (2015) describes human capital and public transfer wealth as the two most quantitatively important forms of “wealth-like objects” that are not captured by measures of market wealth. Catherine et al. (2020) focus on the public transfer wealth; they develop an approach to incorporate Social Security wealth to the measurement of household wealth in the U.S. They find that this addition attenuates increases in wealth inequality since 1989 and reduces top wealth shares compared with other recent literature.

Though these other forms of wealth, due to their non-marketable nature, may be less tangible and difficult to measure, they also represent significant stores of value in Canada. Gu and Wong (2010) produced estimates for human capital wealth in Canada using a lifetime earnings approach; they found that in 2007, the stock of human capital wealth was \$16.4 trillion. By comparison, the (marketable) net worth of the household sector in that same year, as recorded by the National Balance Sheet Accounts (NBSA), was only \$6.0 trillion (Statistics Canada, 2020b).

Social security also represents a significant store of value in Canada. The NBSA includes in its social security funds sub-sector the Canada Pension Plan (CPP) and Quebec Pension Plan (QPP) (Statistics Canada, 2018c). At year-end 2019, the net worth of this sector was valued at \$0.5 trillion (Statistics Canada, 2020b). The NBSA does not accord this net worth to the household sector, but rather to the general government sector. Other social protection “pay-as-you-go” programs, such as federal Old Age Security (OAS) and the Guaranteed Income Supplement (GIS), are not included in the NBSA’s social security sub-sector because those programs do not hold accumulated assets; however, even these transfer programs arguably constitute a form of wealth for households (Catherine et al., 2020).

⁵ Canada’s GDP at market prices in the fourth quarter 2019 was \$2.3 trillion (Statistics Canada, 2020a).

⁶ The concept of a “family” in this report is equivalent to the concept of a “family unit” in Statistics Canada (2018b). This includes economic families and or a person not in an economic family (unattached individual).

Statistics Canada (2018b) defines an economic family as “a group of two or more persons who live in the same dwelling and are related to each other by blood, marriage, common law or adoption.” It defines a person not in an economic family as “a person living either alone or with others to whom he or she is unrelated, such as roommates or a lodger.”

⁷ This wealthiest observation in the 2016 SFS PUMF represents 965 economic families in the general population.

⁸ This analysis is based on Statistics Canada’s Survey of Financial Security Public Use Microdata, 2016, which contains anonymous data collected in the Survey of Financial Security. All computations on these microdata were prepared by the Parliamentary Budget Officer (PBO). The responsibility for the use and interpretation of these data is entirely that of the PBO.

⁹ For more information, see the Survey of Financial Security: Public Use Microdata User Guide, 2016 (Statistics Canada, 2018b).

¹⁰ For more information, see the Canadian System of Macroeconomic Accounts User Guide (Statistics Canada, 2018c).

¹¹ 2016 Q4 is the quarter corresponding most closely to the 2016 SFS collection period. According to Statistics Canada (2019b), the 2016 SFS was collected between 9 September 2016 and 6 December 2016. In addition, as stated in the main text, CB’s Richest People List was also published in 2016 Q4 (December 2016).

¹² For more information, see CB’s Rich 100 methodology (Canadian Business, 2006).

¹³ Auten & Splinter (2019) demonstrate the importance of making assumptions regarding the evolution of family composition when estimating top income shares over time. The authors’ data shows differential changes to family composition across the income distribution (e.g., outside the top of the distribution, there is a declining marriage rate, declining family size, and increasing numbers of single-parent households). All things being equal, such differential changes to family composition over time can be expected to cause changes in the distribution of income among families. It would not be surprising to find that differential changes to family composition can also affect top wealth shares.

¹⁴ Growth in the number of families was approximated by the growth rate in the population reported in Statistics Canada’s Quarterly Demographic Estimates between 2016 Q4 and 2019 Q4. The approximation was necessary because the number of economic families in 2019 Q4 was not available at the publication date. The annual growth rates of the population and of the number of economic families have been within 0.3 per cent of each other since 2012.

¹⁵ Statistics Canada (2019a) provides an excellent exposition of the conceptual differences between the SFS and the NBSA.

¹⁶ Financial assets were calculated using employer pension plans valued on a termination basis, rather than a going concern basis. Statistics Canada (2018b) provides a helpful description of the distinction between the two valuation methods.

¹⁷ Variables were placed according to the mapping presented in Statistics Canada (2019a), except that PBO retained the value of collectibles in the SFS PUMF.

¹⁸ Vermeulen (2016) posits that this adjustment procedure tends to disproportionately increase the wealth of wealthier households, since national wealth surveys tend to underreport financial assets and financial assets represent a higher share of richer families' portfolios relative to poorer ones. This dynamic also occurs in the Canadian data.

¹⁹ An approach that leverages data from household wealth surveys and rich list data to estimate the top tail of the family wealth distribution is used, among others, in Davies (1993), Bach et al. (2014), Bach et al. (2015), Vermeulen (2016), Davies et al. (2017), Vermeulen (2018), Chakraborty et al. (2019), and Davies and Di Matteo (2020).

²⁰ Vermeulen (2018) develops a Monte Carlo study to demonstrate the utility of adding rich lists when estimating top tail. The results show that the addition of a rich list to survey data in the regression to estimate Pareto parameters causes the interpolated wealth tail to be estimated with an upward or downward bias of only 0.01.

²¹ Davies and Di Matteo (2020) provide a helpful discussion on the differences between the Forbes list and Canadian Business' Richest People List and present a comparison of the entries from each list.

²² Bach et al. (2014), Bach et al. (2015), and Davies and Di Matteo (2020) similarly undertake rich list cleaning procedures before incorporating rich list data into national wealth survey microdata.

²³ Economic families with negative net worth in the SFS PUMF are not presented in Figure A2-1. According to the SFS PUMF, there were 878,482 economic families with negative net worth in 2016.

²⁴ In non-formulaic terms, the Pareto power law as applied to the family wealth distribution asserts that the wealth of the n th wealthiest family in the population is inversely proportional to its rank.

²⁵ Canadian Business (2006). The methodology was retrieved using the internet archiving website "Wayback Machine".

²⁶ Though the SFS Master File is not top-coded, the weighting procedure of the survey methodology may reduce the weights of some high-net-worth families even in the Master File. Statistics Canada (2018b) discloses, as part of the weighting procedure, that "influential observation are [...] identified, and weights are reduced for a small number of extreme observations."

²⁷ Vermeulen (2018) explains that it is unclear where the Pareto-distributed top tail of the wealth distribution starts. He addresses the uncertainty by presenting estimates using six different w_{min} thresholds ranging from €500,000 to €10 million.

²⁸ Disclosed to PBO in correspondence with analysts from the SFS Team at Statistics Canada.

²⁹ The 2016 SFS PUMF includes 638 observations with wealth greater than \$3 million. By contrast, no country in Vermeulen (2018) using geographic or income-stratified geographic information to over-sample high-net-worth families had more than 100 observations with wealth greater than €2 million.

³⁰ This value of α falls within Davies and Di Matteo's (2020) range of α estimates to perform top tail imputation on historical Canadian wealth survey data.